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# Getting Started:

First, we started a git repo and created directories with file structures that could be useful in our project. We may not end up using all those files, but this structure will be helpful in enforcing modularity into our program.

A screenshot of a computer program

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Before we start any actual coding, we made sure to review how the actual LU Decomposition works, and just see how in general a set of linear equations is transferred into a matrix that’s to be decomposed and solved using LU Decomposition.

# Step-by-step walk through of the algorithm.

(Making sense of the program we’re making): Those examples will be used to code our initial algorithm and verify its working as expected.

## Example (1)

## Step1: System of Linear Equations

Let’s start with the following system of equations:

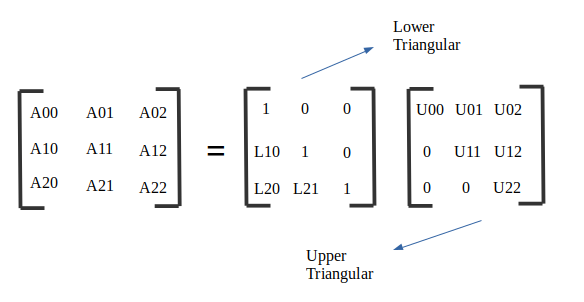
­­­

## Step2: Transforming into Matrix Form:

This system can be represented in matrix form as Ax=b, where:

## Step3: LU Decomposition

Next, we decompose A into L and U, where L is a lower triangular matrix and U is an upper triangular matrix.



In our example:

(Step by step derivations of the LU matrices in the appendix)

## Step4: Solving Ly = b for y

Given L and b, we solve for y (which is an intermediate vector, not the final solution).

Using forward substitution.

## Step5: Solving Ux = y for x

Now, with U and y, we can solve for x using backward substitution.

## Step6: Verifying the answers:

# Example (2):

## Step1: System of Linear Equations

## Step2: Transforming into Matrix Form:

## Step3: LU Decomposition

## Step4: Solving Ly = b for y

Given L and b, we solve for y (which is an intermediate vector, not the final solution).

Using forward substitution.

## Step5: Solving Ux = y for x

Now, with U and y, we can solve for x using backward substitution.

## Step6: Verifying the answers:

# Coding a simple serial program to do this calculation:

To get started, we will code a simple C code implementing this algorithm, then we’ll use those same examples above to verify the correctness of that algorithm. Next, we’ll improve the algorithm to work on bigger matrices, and will use pthreads.

Code structure:

A screenshot of a computer program

Description automatically generated

The matrix is defined in the main here, as a next step, we’ll have It defined in a separate file or function. (The complete code is in appendix 2).

A black screen with white text and numbers

Description automatically generated

# Testing the program:

First example:

The first example we tried was this matrix:

And the answer was:

When we give the program this matrix:

A black background with colorful text

Description automatically generated with medium confidence

We get:

A screen shot of a computer

Description automatically generated

Which is the expected solution.

Second example:

And the answer was:

When we give the program this matrix:



We get:

A screen shot of a computer

Description automatically generated

Which is the expected solution.

We then made the program more structured but adding the matrices in the /matrices directory as text files passed to the program using the command-line terminal as the first argument.

For example:

> ./bin/simple matrices/3x3\_1.txt

Solution:

x[0] = 0.250000

x[1] = 2.000000

x[2] = 1.500000

and

> ./bin/simple matrices/3x3\_2.txt

Solution:

x[0] = 1.000000

x[1] = 2.000000

x[2] = -1.000000

Running the program serially:

# Next steps:

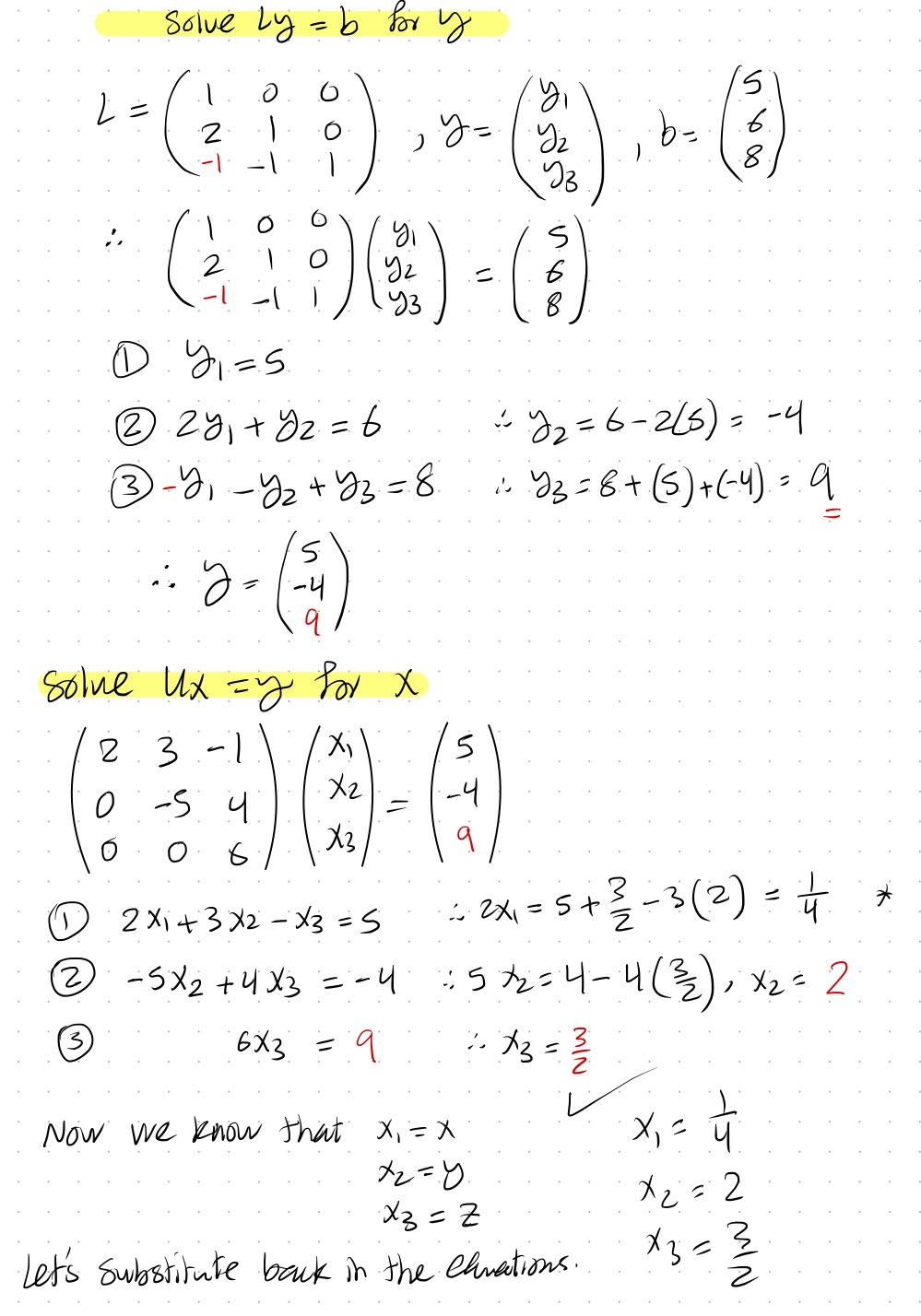
(increase the size of the test matrix)

(Enhance the parallel programming implementation of the program.

# Appendix – 1 (Step by step solution of example 1 using LU Decomposition)

A group of math equations

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A white paper with black text

Description automatically generated

# Appendix –2 (Basic Example code to solve a 3x3 matrix using LU Decomposition)

// Author: Mohamed Ghonim

// Created: 02/18/2024

// Last Modified: 02/18/2024

// Functionality: Perform LU decomposition and solve a system of linear equations using forward and backward substitution.

// Version: 0.1

#include <stdio.h>

#define N 3 // Size of the matrix (3x3)

// Function to perform LU decomposition

void luDecomposition(double A[N][N], double L[N][N], double U[N][N]) {

int i, j, k;

for (i = 0; i < N; i++) {

for (j = 0; j < N; j++) {

if (j < i)

L[j][i] = 0;

else {

L[j][i] = A[j][i];

for (k = 0; k < i; k++) {

L[j][i] = L[j][i] - L[j][k] \* U[k][i];

}

}

}

for (j = 0; j < N; j++) {

if (j < i)

U[i][j] = 0;

else if (j == i)

U[i][j] = 1;

else {

U[i][j] = A[i][j] / L[i][i];

for (k = 0; k < i; k++) {

U[i][j] = U[i][j] - ((L[i][k] \* U[k][j]) / L[i][i]);

}

}

}

}

}

// Function to solve the equation Ly = b

void forwardSubstitution(double L[N][N], double b[N], double y[N]) {

for (int i = 0; i < N; i++) {

y[i] = b[i];

for (int j = 0; j < i; j++) {

y[i] -= L[i][j] \* y[j];

}

y[i] = y[i] / L[i][i];

}

}

// Function to solve the equation Ux = y

void backwardSubstitution(double U[N][N], double y[N], double x[N]) {

for (int i = N - 1; i >= 0; i--) {

x[i] = y[i];

for (int j = i + 1; j < N; j++) {

x[i] -= U[i][j] \* x[j];

}

// No division by U[i][i] since U[i][i] = 1

}

}

int main() {

double A[N][N] = {{2, 3, -1}, {4, 1, 2}, {-2, 2, 3}};

double b[N] = {5, 6, 8};

double L[N][N] = {0};

double U[N][N] = {0};

double y[N] = {0};

double x[N] = {0};

luDecomposition(A, L, U);

forwardSubstitution(L, b, y);

backwardSubstitution(U, y, x);

printf("Solution: \n");

for (int i = 0; i < N; i++) {

printf("x[%d] = %f\n", i, x[i]);

}

return 0;

}